Instability of Laminar Wall Jets along Curved Surfaces

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THIS note presents some results of a linearized analysis of the Taylor-Görtler instability in laminar wall jets along curved surfaces. Interestingly enough, a wall jet (due to the nature of its mean profile) exhibits this instability on convex as well as concave walls.

Following convention, the region between the wall and the maximum in the velocity profile is defined as the inner flow, while the region beyond is designated the outer flow. When the flow is along a concave surface, the inner flow is unstable. This is designated Type I instability. Flow along a convex surface results in an instability of the outer flow. This is named Type II instability.

Theory

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$$(d^2u/d\eta^2) - [V_0R_d + K]\,(du/d\eta) + [B^2 - A^2 - R_d(BU_0 - dV_0/d\eta) + K\eta B(2B - U_0R_d)]u - [R_d(dU_0/d\eta) + K(2B - U_0R_d)]v = (B/2)C(1 + K\eta) \qquad (1)$$

$$(d^2v/d\eta^2) - [V_0R_d + K]\,(dv/d\eta) + [B^2 - A^2 + 2K\eta B^2 - BR_dU_0(1 + K\eta) - R_d(dV_0/d\eta)]v - [2K(R_dU_0 - B)]u = \tfrac{1}{2}(dC/d\eta) \qquad (2)$$

$$(d^2w/d\eta^2) - [V_0R_d + K]\,(dw/d\eta) + [B^2 - A^2 + 2KB^2\eta - BR_dU_0(1 + K\eta)]w = -(A/2)C \qquad (3)$$

$$Bu + KBu\eta + (dv/d\eta) + Aw - Kv = 0 \qquad (4)$$

where u, v, w are the dimensionless fluctuations of velocity in the x, y, z directions, respectively, and C is the dimensionless pressure fluctuation. η is the dimensionless vertical (y) coordinate. U_0 and V_0 are the dimensionless streamwise and vertical velocities, respectively, of the primary flow. A, B and K are the dimensionless wavenumber, amplification parameter, and curvature, respectively, concave curvature being denoted by a positive K. The scaling length and velocity used are those proposed by Glauert[2] in a similarity analysis of the laminar walljet. Consequently, this permits direct use of his similarity solutions for the mean velocities U_0 and V_0 in the stability calculations. The Reynolds number R_d then becomes $(U_s x/\nu)^{1/4}$ where U_s is a velocity related to the outer momentum flux, ν the kinematic viscosity, and x the distance downstream from a virtual origin. The boundary conditions applicable are $\eta = 0$, $u = v = w = 0$, $\eta \to \infty$, $u, v, w, \to 0$. Additionally, the continuity Eq. (4) for the fluctuations yields the auxiliary boundary condition $\eta = 0$, $dv/d\eta = 0$.

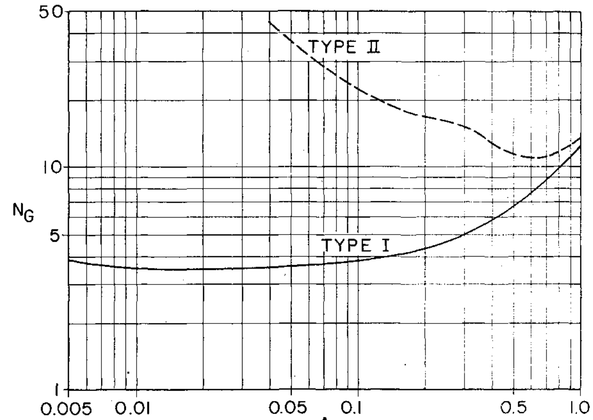

Fig. 1 Neutral stability curves.

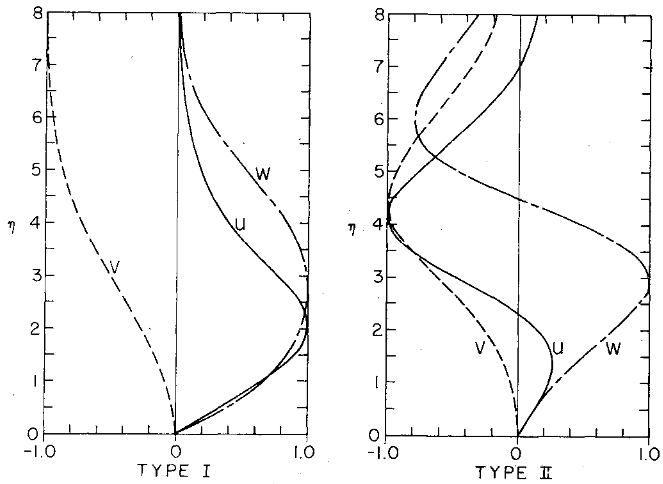

Fig. 2 Eigenfunctions at critical conditions.

The equations are homogeneous with homogeneous boundary conditions and therefore constitute an eigenvalue problem. The information sought is the variation of K with A, all other parameters remaining fixed. A multiple shooting method using a Hamming predictor-corrector integration formula was used in the numerical solution procedure. To restrict the range of integration to a finite interval, asymptotic solutions valid in the far field region were derived and matched to the numerical integration at $\eta = 10$. Details of the method are described in Conte.[3]

Results and Discussion

The results are presented in terms of the Görtler number defined here as $R_d\sqrt{K}$. They were found to be independent of Reynolds number, a result also found by Smith.[1] The neutral stability curves ($B = 0$) for the Type I and Type II instabilities are shown in Fig. 1. The critical values of Görtler number and wavenumber for the Type I instability are considerably lower than those of Type II. This is most probably due to the action of the vertical velocity component of the primary flow in limiting the size of the Type II disturbances. Figure 2 demonstrates the comparatively stronger penetration of the v perturbation in the Type I instability, although eventually it decays to zero.

Received March 3, 1975; revision received July 7, 1975. Work partially supported by National Research Council of Canada, Grant No. A-8846.

Index category: Boundary-Layer Stability and Transition.

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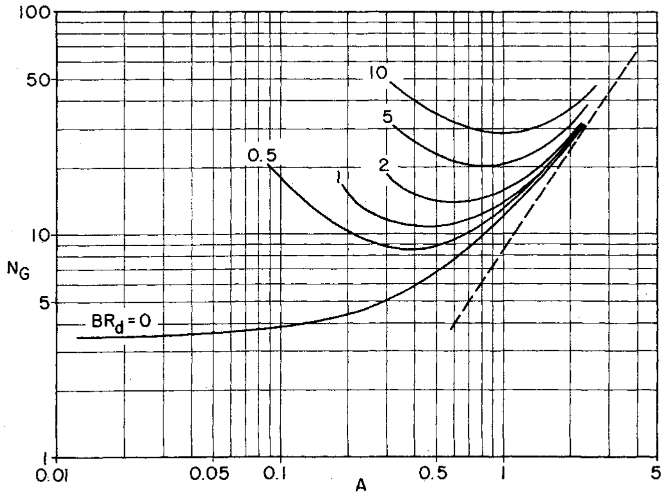

Fig. 3 Amplification curves for Type I instability.

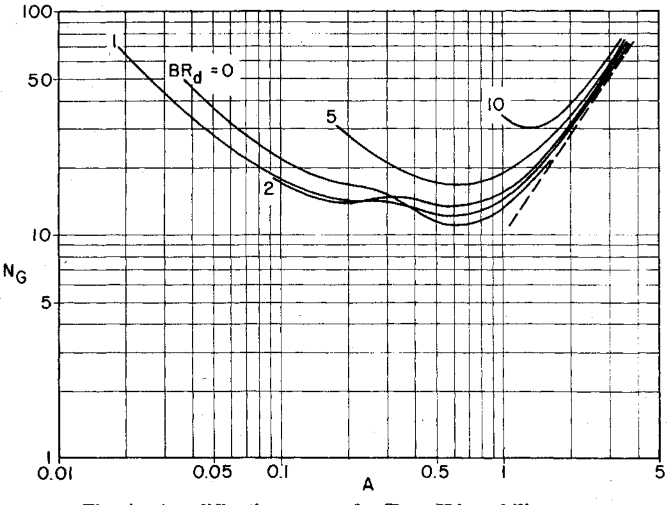

Fig. 4 Amplification curves for Type II instability.

Figures 3 and 4 are calculated spatial amplification curves for the Type I and Type II perturbations, respectively. By eliminating the characteristic length δ from the definition of the Görtler number and the dimensionless wavenumber ($=\alpha\delta = 2\pi\delta/s$, where λ is the lateral wavelength), one obtains $N_G^2/A^3 = U_s^2\lambda^3/8\lambda^2\nu^2 R_0$, where R_0 is the radius of curvature of the plate. If U_s, R_0, and λ remain constant with increasing x, then N_G^2/A^3 becomes a constant C_0. This is represented by the dashed line of gradient *3/2* on the log-log plots of Fig. 4. When the constant C_0 has a value at which the straight line is below the neutral curve, the vortices with the wavelength λ corresponding to C are damped spatially owing to viscous effects. When $C \geq 72.3$ for the Type I instability or ≥ 97.8 for the Type II instability, the straight line touches or cuts the neutral curve and the walljet enters an unstable region. These values of C are not too different from the corresponding value ($C \sim 78$) calculated for the laminar boundary layer and the asymptotic suction profile from the results of Smith[1] and Kobayashi,[4] respectively.

It is rather interesting that both the Type I and II amplification curves appear to be tangential to the line of *3/2* gradient. This indicates that vortex wavelength in this region is highly selective and spatial amplification would occur at constant wavelength until nonlinear effects became important.

Conclusion

An analysis of centrifugal instability in walljets along curved surfaces has been performed using linear perturbation theory. Instability on concave as well as convex walls has been investigated. For a given Reynolds number as well as radius of curvature, it is found that the walljet is more unstable on a concave wall than on a convex one. The work reported here could also find application in turbulent walljets. Tani[5] has demonstrated the utility of stability calculations of Taylor-Görtler vortices applied to turbulent boundary layers.

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Gravitational Effects on Process-Induced Defects in Single Crystal Silicon

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For the past several years semiconductor manufacturers have been concerned with imperfections that are induced during device fabrication. Considerable effort has been directed toward understanding the causes of these dislocations.[1,2] Various processing steps have been identified as major contributors to the generation of dislocations. During diffusion, thermal gradients sufficient to cause the generation of dislocations are produced.[3] Impurity depositions lead to concentration gradients which produce stresses that result in dislocation creation.[4] It is also known that the thermal mismatch of the oxide at the oxide-silicon interface can lead to the generation of undesirable imperfections in the wafer.[5]

While these steps have been identified as crucial ones in controlling process-induced dislocations, the critical conditions present when the dislocations are generated during each step have not been determined. To understand the exact conditions present when dislocations are generated in any one of these steps requires that one be able to separate the various influencing factors. It is necessary, then, to be able to create the dislocations selectively while maintaining control of all the critical parameters. One critical parameter which has been largely ignored is the gravitational stress in the wafer, which is ever present in earth-bound processing experiments.

This parameter alone leads to important questions concerning possible advantages for space production of semiconductor devices. Previous emphasis has been on semiconductor crystal growth, although starting material quality produced on earth is considerably better than that which can be maintained during earth processing. This paper discusses a series of

Presented as Paper 74-647 at the AIAA/ASME 1974 Thermophysics and Heat Transfer Conference. Boston, Mass., July 15-17, 1974; submitted April 1,1975; revision recieved July 28, 1975. This project supported by NASA contract NAS8-29851.

Index categories: Materials, properties of; Atomic, Molecular, and Plasma Properties.

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